

Long-term debt and hidden borrowing*

Heski Bar-Isaac and Vicente Cuñat[†]

New York University and London School of Economics

June 2009

Abstract

We consider borrowers with the opportunity to raise funds from a competitive banking sector that shares information, as well as from an alternative hidden lender. The presence of the hidden lender allows borrowers to conceal poor results from their banks and, thus, restricts the contracts that can be obtained from the banking sector. In equilibrium, some borrowers obtain funds from both the banking sector and the inefficient hidden lender simultaneously; cross-subsidies arise between different borrowers leading to too little liquidation and limiting the extent of bank lending. Imposing distributional assumptions, we fully characterize outcomes and conduct comparative statics exercises. We generalize the model to allow for a partially hidden lender and obtain qualitatively similar results.

Keywords: Long-term debt, hidden borrowing, debt contracts, adverse selection

JEL Codes: D82, D14, G21, D86

1 Introduction

Firms and consumers have access to different sources of borrowing. Loans may differ in their seniority, covenants, and interest rates. This may induce an apparent pecking order

*We thank Ken Ayotte, Alberto Bisin, Patrick Bolton, Leonardo Felli, Charles Goodhart, John Moore, Tomasz Piskorski, Larry White and audiences at the Econometric Society World Congress, ESSFM Gerzensee, FMG, and Columbia for their comments. Cuñat thanks the BBVA Foundation for financial support.

[†]<heski@nyu.edu> Department of Economics, Stern School of Business, NYU, 44 West 4th street 7-73, NYC, NY 10012 USA; and <v.cunat1@lse.ac.uk> UPF, Room A450, Department of Finance, London School of Economics, Houghton Street, London WC2A 2AE, U.K.

between them. However, loans may, also, differ in the extent of informational opacity with respect to other lenders. While some lenders perfectly share information—through a public credit registry, for example—other lenders may have no involvement in such information sharing. Borrowers may choose more opaque loans in order to conceal information from others.

This paper investigates the consequences of the presence of opaque loans in terms of the types of loans offered, the amount of credit extended, and welfare. Our results show, first, that the presence of an opaque lender limits the contracting options of other lenders: If all lenders perfectly share information, loans induce borrowers to reveal their solvency at all times by having interest loans that are highly responsive to repayment schedules. However, if borrowers can secretly obtain funds, loans becomes less responsive to repayments which might reflect a borrower’s access to alternative loans in addition to his credit-worthiness. A borrower may simultaneously access both opaque and transparent loans, even when more opaque loans may be more costly in terms of higher interest rates. Second, the presence of opaque loans generates cross subsidies between borrowers that lead to inefficient liquidation policies. Third, opaque lending limits the amount of lending as the induced contractual limitations reduce the surplus that lenders can extract.

Our results provide one explanation for the empirical observation that borrowers appear to borrow from apparently costly lenders without fully exhausting cheaper sources. Firms use costly trade credit and personal loans from the owner before exhausting their credit lines and while having free collateral.¹ On the consumer side, Gross and Souleles (2002), for example, report that in a large sample of credit card holders, almost 70% percent of those borrowing on bankcards have positive housing equity. A possible rationale for this behaviour is that by using alternative sources of borrowing that are not perfectly observable to their main lenders, borrowers can conceal their liquidity shocks.²

For example, missing a repayment can trigger a renegotiation with the bank and lead to a higher future interest rate. This reflects the bank’s renewed assessment of the borrower’s ability to repay. An effort to renegotiate the loan may be costly for the borrower because

¹For example, in the 1998 National Survey of Small Business Finance (NSSBF), among the firms with bank debt not exceeding the value of their land (a conservative estimate of firms with free collateral), 14.7% used trade credit and 13.5% used lines of credit.

²Other explanations have been posited to explain this apparent puzzle; for example, Laibson et al. (2003) calibrate a model of life-cycle borrowing with time-inconsistent preferences, and Bertaut, Haliassos and Reiter (2009) discuss a model of separate mental accounts. The results of this paper assume fully rational consumers and need not contradict such explanations, but can be seen as complementary to them.

it would reveal information about current and future cash flows. In order to avoid this penalty, an entrepreneur might borrow from elsewhere, taking a personal loan, for example, to conceal the bad news that the enterprise has suffered a negative shock. In turn, this makes missing a payment even worse news, as it reflects a negative shock so large that it is prohibitively costly to conceal. The resulting overall cost of renegotiation may be sufficiently high that the financier would repossess the asset or foreclose following a missed payment.

We illustrate the interaction between publicly-observable and hidden borrowing more formally in a two-period model, where agents have access to an investment project that yields cash flows correlated across time. They can fund the project through two sources: a competitive banking sector that shares information; and an opaque lending sector. Banks are senior claimants and seek to obtain information regarding borrowers through interim payments. While most of our discussion views banks as providing flexible long-term (two-period) financing, one could also interpret the banking sector as providing a sequence of short-term loans.

Our first results show that if the alternative source of borrowing is sufficiently inefficient (or is absent), banking contracts will achieve first-best. By rewarding higher interim payments with lower future interest rates, the optimal contract gives borrowers incentives to reveal their intermediate cash flows perfectly. However, with a viable alternative hidden lender, a borrower might be tempted to borrow from that source in order to disguise her type. The original lender in the banking sector anticipates this possibility. In general, this will lead to a more limited menu of repayment schedules in the optimal contract. Further, we show that some borrowers borrow from the opaque sector to make the interim repayment. Thus, in equilibrium, these borrowers are simultaneously borrowing from both the banking and the opaque sectors. Since information is suppressed, banks cannot easily distinguish good from bad projects, liquidation decisions are inefficient, and some projects continue for longer than they should. This raises the effective cost of borrowing.

Additional results can be achieved if we impose a distributional assumption on creditors' types. Assuming that types are uniformly distributed allows us to fully characterize equilibrium, and, in that case, we can show that the unique equilibrium results in only a single level of interim payment observed in the banking sector. We also allow for ex-ante opportunity costs for a potential borrower to initiate a loan and for partially hidden lending. We perform comparative statics exercises: when opaque lenders are less efficient or it is easier for banks to observe loans from alternative lending sources, bank contracts

are more flexible, interest rates are contingent on interim repayments, and the volume of transparent lending initiated increases.

Related Literature and Supportive Evidence The empirical predictions from the comparative statics exercises are consistent with historical and international evidence (see, in particular, the edited volume Miller, 2003; Hunt, 2006; Jentzsch, 2007; and Jappelli and Pagano, 2006). Consider, for example, the effect of more easily available information for creditors on loan initiation, which is perhaps the outcome that is easiest to observe: In Miller (2003), Barron and Staten, reporting on lessons drawn from the history of credit reporting in the U.S., discuss the remarkable growth in credit and argue that such growth appears efficient inasmuch as default rates have not grown. They state: “The U.S. credit reporting environment is the foundation for this remarkable combination of widespread availability and low default rates.” (p.287). Cowan and De Gregorio (2003) show evidence from Chile that information sharing increased the volume of lending. Jappelli and Pagano (2002) provide evidence that bank lending is higher, and credit risk lower in countries where lenders share information; and Brown, Jappelli and Pagano (2009), in an investigation of firms in Eastern Europe and the former Soviet Union, “show that information sharing is associated with improved availability and lower cost of credit to firms” (p. 1). It is worth noting, however, that this evidence is consistent with alternative, related models of information sharing (Padilla and Pagano, 1997, Jappelli and Pagano, 1993 and, more broadly, see the discussion in Jappelli and Pagano, 2006). However, this previous literature has discussed the complexity of contracts very little. Our model suggests that higher availability of creditor information should lead to debt contracts with more flexible repayment terms and schedules, rather than, say, fixed repayment levels at fixed dates. This seems consistent with anecdotal evidence on the development of mortgage contracts in the UK, for example, where recent years have seen a growth in flexible (or “lifestyle”) mortgages.

In this paper, the banking sector cannot write contracts that make payments depend on the amount borrowed from the hidden lender. This is a natural consequence of the assumption that the banking sector cannot observe borrowing from the hidden lender. This paper is, therefore, related to a growing literature on non-exclusive contracts and hidden savings.

Our focus on *different lending sectors* that vary in the information that they have and the simple comparative statics analysis that this allows, distinguishes our paper from the literature on exclusivity. For example, there are models of non-exclusivity with simultane-

ous contracting (Bisin, and Guaitoli, 2004, or Jaynes, 1978 and Arnott and Stiglitz 1991 in the context of insurance markets) or with sequential access to loans (Bizer and DeMarzo, 1992) and with financial intermediaries who are *ex-ante* identical.

In the optimal contracts that we characterize, interim payments provide useful information that can allow for more efficient outcomes. This mirrors observations in Allen (1985) and Dionne and Lasserre (1985). Hidden borrowing or savings (as in Cole and Kocherlakota 2001) can, therefore, create inefficiency in these environments by reducing the information available from interim payments.

A feature of our analysis is that we vary the cost of borrowing from the hidden source. Allen (1985) and others focus on the case where this cost is equal to the social planner's rate.³ Innes (1990), in order to generate monotonicity in repayment schedules, considers the case where money can be repaid immediately so that the cost of borrowing is essentially zero.

Finally, a key element of the model is that a lender may not perfectly observe all the loans that a borrower may hold. Empirically, this is certainly the case. For example, although information sharing takes place through credit bureaus, there are many lenders who choose neither to pay for access to credit bureaus nor to provide information to them. Trade credit, informal, black-market lending, and personal loans to entrepreneurs subsequently used in their firms are clear examples. Further examples include consumer credit, store credit, payday lenders, and other sources that do not participate in organized information-gathering credit bureaus, both in developing countries and elsewhere, both currently and historically.⁴ For instance, Barron and Staten (2003) highlight that in some Latin American countries, there are "comprehensive credit histories on consumers but only on loans held by commercial banks" (pp.273-4). Note, further, that, even when a lender has access to a credit bureau, the costs associated with accessing and processing the relevant information may lead lenders to obtain and use this information only in particular circumstances. Such circumstances would include the loan-approval stage, missed payments, and renegotiation; otherwise, there is unlikely to be continual monitoring. In this paper, we simply take it for

³The general model of Doepke and Townsend (2004), as illustrated in their example in Section 7.1, allows for this more general interest rate; however, as in Cole and Kocherlakota (2001) and Ljungqvist and Sargent (2003), they consider hidden saving and insurance rather than hidden borrowing and focus on numerical rather than analytical solutions.

⁴For example, in the U.S., payday lenders do not share information with banks (Elliehausen and Lawrence, 2001; Mann and Hawkins, 2007). However, it has been shown that their presence alters the borrowers' payment of other loans. In particular, mortgage delinquency after an aggregate liquidity shock is significantly lower in areas where there are payday lenders (Morse 2007).

granted that some types of borrowing are not commonly observed by all lenders.

2 The Model

Although the underlying economic mechanisms have wider applicability, we focus the model on the particular example of a small business that is raising funds for a capital investment for a project that will generate an interim and a final return. These pay-outs are positively correlated and, so, at the interim stage, in particular, there is additional information that is useful for assessing creditworthiness. The firm has access to both a competitive banking sector and a hidden lender. One can think of the hidden lender as a personal loan to the entrepreneur secretly diverted to the firm.

2.1 Set-up

We introduce a two-period model to consider the interaction between alternative sources of borrowing: a transparent banking sector and an opaque hidden lender (or lending sector).

2.1.1 Lending Sectors

In the transparent sector, credit is provided by a continuum of agents that we call banks. Banks are risk-neutral deep pockets, and there is competition among them. Banks share information, and so the borrowing position of any borrower with a bank is perfectly observable and verifiable among all banks. We normalize the gross riskless market interest rate of this banking sector to one. The principal assumptions about the banking sector can be summarized as follows:

Assumption 1: The total amount of loanable funds in the banking sector exceeds demand.

Assumption 2: A borrower can repay her outstanding balance and switch to another bank at any point in time.

Assumption 3: Banks perfectly share the information about the borrower's outstanding loans.⁵

These assumptions guarantee both that banks do not make a profit, on average, and that conditional on the information known at any point in time, every contract offered must break even. In short, there can be no observable cross-subsidies between borrowers.⁶

⁵In particular, this implies that they cannot simply replicate the hidden lender, as they have no means to hide such contracts from other banks.

⁶If a set of borrowers knew and were able to prove to a third party that they were subsidizing other borrowers, they would switch to another bank, leaving their previous bank with only subsidized borrowers and, thus, losses.

In addition to the transparent banking sector, we introduce an alternative opaque lending sector that lends at a flat repayment rate $r > 1$; for now, we take it as exogenous, though we discuss endogenizing this interest rate below.

A key feature of this alternative borrowing source is that it does not share information with the rest of the financial system. That is, the borrowing position of any borrower in the opaque sector is not observable by banks. Further, we model the opaque sector as a junior lender. This is certainly consistent with an interpretation as a concealed loan from the firm owner to the firm.⁷ In our model, lenders exogenously belong to either the banking sector or the opaque sector.

2.1.2 Borrowers

Demand for funds comes from borrowers who require these funds for an investment project and who are heterogeneous in the quality of their projects. They are risk-neutral and maximize total consumption across periods. These assumptions may not be crucial for the qualitative insights; however, they are convenient in characterizing a unique equilibrium outcome.

The timing of the model is as follows:

At $t = 0$, each borrower does not know her type. In order to raise D units of funding necessary to invest in the project, the borrower can costlessly search across banks for a menu of first- and associated second-period debt repayment schedules $\{p, q(p)\}$. Second-period payments may be contingent on first-period ones.⁸

At $t = \frac{1}{2}$, each borrower learns the type of her project, which is parametrized by α , where α is distributed on $[0, 1]$. At this point, the borrower can either costlessly liquidate the project for D and fully repay the loan or continue with the project.⁹

At $t = 1$, borrowers realize a cash flow α that corresponds to their type. They can choose to borrow d from the opaque lending source that is hidden from the banks. The opaque lender is junior to the bank loan, and banks do not observe d . Borrowers can use these funds to either consume or choose one of the repayment schedules from the menu and repay p to the bank.

⁷In terms of seniority, it is also consistent with trade credit or credit cards. Other types of hidden lending, including black-market lending, may be more ambiguous with respect to seniority.

⁸Note that costless search and a competitive banking sector is outcome-equivalent to the borrower having full bargaining power and proposing the schedule to a single bank.

⁹We model this option to stop the project as a costless liquidation at a very early stage; but supposing that the agent were able to recover a sufficiently large salvage value at an early stage would generate similar qualitative results.

At $t = 2$, the project is successful and delivers $B + \alpha$ with probability ν . Otherwise, the project fails and delivers only α . In both cases, seniority of debt is such that the borrower repays $q(p)$ to the bank first and then repays opaque lenders up to rd . The borrower consumes all the remaining funds.

Note that, in the interim period, a borrower has the option to consume anything left over and optimally chooses to do so. Any savings may end up being used to repay debts, and the borrower prefers to consume early for sure to saving and then consuming, with some probability, later on. In particular, it follows that residual interim income is never used to repay future debts.

The parameter α represents the creditworthiness of the borrower since the expected final cash flow of the project is positively correlated with its interim cash flow. Note that, overall, a project of type α generates a net present value of $-D + \alpha + \nu(B + \alpha) + (1 - \nu)\alpha = -D + \nu B + 2\alpha$. In particular, the best potential project, a project of type $\alpha = 1$, generates an expected net present value $Y \equiv 2 + \nu B - D$. Low values of Y suggest (though, obviously, depending on the distribution of types) that a high proportion of projects are inefficient. In particular, $Y \leq 0$ implies that no projects should be funded, while $Y \geq 2$ implies that all projects are efficient and should be funded. With intermediate values of Y , only projects with $2\alpha \geq 2 - Y$ are efficient.

The following diagram summarizes both the borrower's actions and the payoffs required and generated by the investment project.

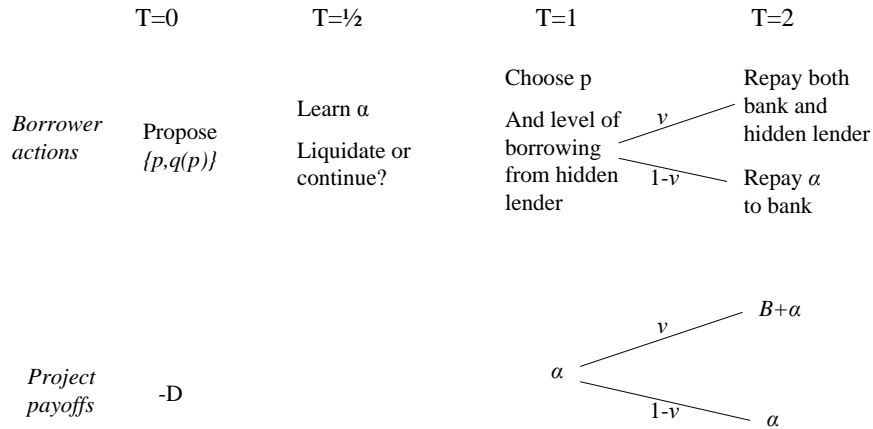


Figure 1: Summary of timing

Borrowers and lenders are risk-neutral, and every agent seeks to maximize the sum of her first- and second-period incomes.

2.2 Further Assumptions

In this section, we add three auxiliary assumptions that help to simplify the analysis. The first assumption ensures that contracts are renegotiation-proof; the second assumption precludes unlimited borrowing; and the third imposes parametric restrictions that rule out uninteresting cases.

Assumption 4: Banks weakly prefer renegotiation-proof contracts.

In the absence of Assumption 4, more-general contracts could arise in period 0, but renegotiation would lead to the same outcomes characterized by the model.¹⁰

The renegotiation-proof condition is effectively equivalent to an exclusivity-proof contract—that is, a contract that guarantees that at any point in time the borrower does not want to switch to another bank (in this sense, the contracts are exclusive proof between banks, as in Rampini and Bisin 2006). This highlights our focus on contracts that are exclusivity-proof within the banking sector, but where exclusivity cannot always be enforced with respect to the hidden lender.

We make the following assumption that ensures that the borrower does not borrow to finance current consumption, but only for the sake of investment.

Assumption 5: A borrower cannot owe more than she can possibly repay in the best possible state (that is, no more than $B + 2\alpha$).

This assumption can be understood as a “no fraud” condition. For example, it might be appropriate if borrowers could be punished beyond limited liability if it were found (perhaps with some probability) that they did not intend to repay in any possible state of the world. This is a sensible borrowing limit since most legal systems allow for punishment above limited liability (i.e., prison or personal liability) whenever a borrower takes a loan that she does not intend to repay even in the best possible situation.¹¹

¹⁰For example, banks could offer a repayment schedule with a common first period interest rate that would make them break even on an average borrower. Then, the schedule could have an extremely high second period interest rate that would surely be renegotiated at $t = 1$ in view of the information revealed by p . Since banks are competitive, the new renegotiated interest rate would make banks break even, conditioning on this new information. The outcomes and payoffs under renegotiated and renegotiation-proof contracts are identical, so the role of Assumption 4 is to emphasize the long-term nature of the contract.

¹¹Note that such a borrowing limit requires the payoff to become verifiable in case of default. We believe, that it is plausible that if the project fails, triggering liquidation and investigation, α becomes verifiable but in the absence of a liquidation proceedings, it is not. Introducing a small verification cost in Period 2, in the spirit of the costly state verification literature (Townsend, 1979; Gale and Hellwig, 1985), would not

Assumption 5 ensures that borrowing from the hidden source in order to consume will not occur. Borrowing from the hidden sector to consume and repay in the good state is inefficient. Therefore, borrowing to consume would be worthwhile only if the borrower intended to default for sure, and Assumption 5 precludes this possibility. Note, however, that in the results characterized below, this borrowing limit is not binding, and so, although it is significant in ruling out borrowing for the sake of consumption, it plays no further role in the characterization of the equilibrium.¹²

Finally, we make parametric restrictions that preclude some trivial and uninteresting cases.

Assumption 6: $D > 2$ and $0 < Y < 2$

The first restriction ensures that no borrower can repay for sure; the second restriction ensures that all types of borrowers will default to a different extent if the project is unsuccessful (so, from the point of view of lenders, they really are different types) and, in particular, some projects are efficient and some are not.

3 Equilibrium

The feasible strategies for the banks are menus of repayment schedules $\{p, q(p)\}$ at $t = 0$. The borrower chooses which offer to accept, if any. If the borrower refuses all offers, the game ends; otherwise, having accepted an offer, the borrower has to decide whether to pursue the project at $t = \frac{1}{2}$ or to liquidate. Finally, the borrower has to decide which schedule and first payment from the menu to choose, funding any shortfall for the first payment by borrowing from the hidden source.

The equilibrium configuration turns out to depend crucially on whether the interest rate at which the hidden sector lends r is above or below a threshold $\frac{2-\nu}{\nu}$. Above this threshold, opaque lending is too expensive to be used to conceal a bad realization of α making it irrelevant, whereas below this threshold, it plays a role. We prove this and separate these two cases in the discussion that follows.¹³

affect the qualitative results.

¹²Even though the “no fraud” condition leads to a different borrowing limit across borrowers, banks cannot use this to separate them, forcing them to reach this limit on their first payment p . As p increases, the second payment q becomes negative before reaching the borrowing limit, giving the borrowers additional borrowing capacity. Therefore, if hidden borrowing is used only for concealment purposes, the “no fraud” condition is never binding. It can bind only if the purpose of borrowing is consumption.

¹³The intuition for this threshold is as follows: Under perfect separation, using one unit of hidden borrowing to repay the banks generates a reduction in the outstanding balance of 1 plus an improvement by $1 - \nu$ in the banks’ expectation of future payoffs. This benefit has to be compared with the cost of

Throughout, the exogenous interest rate r can be thought of as a measure of the degree of inefficiency of the opaque sector. The break-even rate for r is $\frac{1}{\nu}$, and this would be the endogenous rate for the opaque sector if there were no other frictions or inefficiencies. Regardless of the amount borrowed, the opaque lender will always be repaid if the good state is realized and will always face default in the bad state.¹⁴ However, whether we think of the opaque lender as trade credit, a credit card, personal loans to an entrepreneur, or an informal lender, it is reasonable to believe that the interest rate charged could be above this break-even rate—for example, if there are other uses or users of this source of lending. We, therefore, study situations in which $r > \frac{1}{\nu}$.

3.1 Very inefficient opaque lender

In this section, we explore the implications of a very inefficient opaque sector. In particular, we explore the resulting equilibrium when the interest rate r is bigger than $\frac{2-\nu}{\nu}$. In this range, borrowing from the opaque lender is so expensive that it is irrelevant. Here, we discuss an equilibrium where there is full separation among those types that borrow—that is, each different type repays the banking sector a different interim payment, and there is no borrowing from the opaque sector. The intuition is that this outcome leads to efficient liquidation decisions (since each type pays exactly what it should) and since competition results in borrowers retaining all the surplus, this is the outcome that they prefer at the ex-ante stage where contracts are determined. We characterize this equilibrium then briefly discuss other equilibria.

Proposition 1 *When the opaque sector lends at a sufficiently high interest rate ($r > \frac{2-\nu}{\nu}$), there exists a fully separating equilibrium where all banks offer the same equilibrium contract. This contract is a contingent one, where the interim payment is equal to the first-period cashflow and the corresponding final payment fully reflects the information implied by the revealed first-period cashflow. Liquidation at $t = \frac{1}{2}$ is at the efficient level.*

Proof. The proof of this and all subsequent results appear in the Appendix. ■

borrowing from the hidden source of $\frac{r}{\nu}$. So whenever $r > \frac{2-\nu}{\nu}$ it does not pay to use hidden borrowing from concealment purposes. Note that the assumption that default probabilities are type independent help us to simplify the analysis here but a similar threshold would arise if they were type dependent.

¹⁴This follows from the seniority of bank debt, the size of the project, and Assumption 5. Note that this is independent of the type of the project, and this is precisely the reason why the information held by the hidden lender is irrelevant to our analysis, as the hidden lender cannot gain from conditioning on the main loan size or its payments.

Lemma 1 *The above equilibrium achieves first-best.*

Note that since the absence of an opaque sector is equivalent to one where the cost of borrowing is infinitely large, it is a trivial corollary of Lemma 1 that the first-best can be achieved if there is no opaque sector.

Formally, beyond the equilibrium described in Proposition 1, there are many other essentially observationally equivalent equilibria, in the sense that the offered menu could include many other redundant $\{p, q(p)\}$ schedules that are never taken up and that have no effect on outcomes (for example, schedules with very high p 's and q 's), or where some banks (that, in any case, earn no expected profits) offer menus that are never taken up. Henceforth, we ignore such equilibria.

3.2 Relatively Efficient Opaque Lender

In this section, we explore the equilibrium outcome when the opaque sector is more efficient—that is when $r < \frac{2-\nu}{\nu}$. Note, in particular, that this regime includes the case where there are no frictions in the opaque sector and r is arbitrarily close to $\frac{1}{\nu}$.

Even though a full characterization of the equilibrium when $r < \frac{2-\nu}{\nu}$ requires specification of the distribution of types, we can still describe some general features of any existing equilibria. In particular, we can determine that there will be some pooling among different types of borrowers with regard to their interim payments. Further, banks are not able to distinguish the different types within a pool; it follows that there will be some cross subsidization between borrowers and, therefore, liquidation decisions that are inefficient.

Before we present a more formal characterization of the outcome when $r < \frac{2-\nu}{\nu}$, we provide a preliminary result on the weak monotonicity of payments with respect to type. Then, we are able to show in Proposition 2 that individual separation cannot be achieved.

Lemma 2 (*Monotonicity of p*) *A borrower that earns a higher interim-period return will pay a (weakly) higher interim repayment. (More formally, for every type $\alpha > \beta$ that does not liquidate, $p(\alpha) \geq p(\beta)$).*

This lemma states that higher types do not make lower interim repayments than lower types do. The proof follows by contradiction: If such a contract were an equilibrium, then it seems natural that there is no cost for a lower type to mimic a higher type by paying less in the interim period (and possibly cutting the interim repayment will save on costly borrowing from the hidden sector). Formally, the result is proved by examining the relevant incentive-compatibility constraints.

This result allows us to prove the following Proposition.

Proposition 2 *When the hidden lender’s interest rate is sufficiently low ($r < \frac{2-\nu}{\nu}$), there cannot be an equilibrium where a continuum of borrowers are able to fully separate. Further, borrowers’ types can be partitioned, with each pool of borrowers paying a different interim payment.*

Again, underlying the proof of this result are the incentive constraints of borrowers to choose the appropriate schedule from the menu. By characterizing each of the incentive constraints (imitating a borrower of a higher type or of a lower type) in different cases, we complete the proofs. The intuition here is again a natural one. If, in some region, two similar types can fully separate, then, by borrowing “a little” from the hidden lender, the lower type can mimic the higher and be better off overall. That is, by borrowing marginally, the borrower can affect the interest rate on infra-marginal outstanding debt.

We can further characterize properties of any equilibrium contract. In particular, we determine that the marginal borrower who does not liquidate cannot be consuming after making her first payment. Let l denote the type that is “just indifferent” between liquidating and continuing the project with the $(p(l), q(l))$ repayment schedule that corresponds to the lowest pool of borrowers.

Proposition 3 *The marginal borrower indifferent between continuing the project and liquidating does not consume in the interim period (that is, $l \leq p(l)$).*

Propositions 2 and 3 imply that when $r < \frac{2-\nu}{\nu}$, if there exists an equilibrium, it must be one in which all borrowers belong to some pool. That is, no borrower is able to fully separate.¹⁵

This is an important result of the model. The presence of a relatively efficient hidden lender restricts the contractual options of the bank, forcing the contract to be less contingent on intermediate payments. As the interest rate of the hidden lender falls, banks find it harder to distinguish between borrowers. Note that, within a pool of indistinguishable borrowers, the interest rates between $t = 1$ and $t = 2$ are the same for all borrowers, regardless of their effective creditworthiness. Higher-quality borrowers cross-subsidize lower-quality borrowers. This is true in all possible pools of borrowers and, in particular, in the bottom pool of borrowers who do not liquidate their projects. Therefore, this leads to inefficient liquidation policies, with too few projects being liquidated.

¹⁵Note, however, that we cannot rule out the existence of multiple pools.

3.3 Full Characterization

We continue to consider the case in which the hidden lender is relatively efficient—that is, where $r \leq \frac{2-\nu}{\nu}$. To progress and give a full characterization of equilibrium, we introduce a specific distributional assumption.

Assumption 7: $\alpha \sim U[0, 1]$

We maintain this assumption throughout the remainder of the paper. The uniform distribution allows for a tractable full characterization of equilibria. Moreover, the assumption that borrower types are uniformly distributed emphasizes the forces described above, insofar as it leads to full pooling: All types that borrow from the transparent sector will choose the same contract from the schedule. Rather than the menu of contracts actually taken up in Section 3.1, borrowing from the transparent sector will entail the same payment p at $t = 1$ for all types who have not liquidated and the same remaining debt q due at $t = 2$ (which will be fully repaid in the good state and only partially repaid—to an extent that depends on type—in the bad state).

Proposition 4 *When the lending rate from the hidden sector is sufficiently low ($r \leq \frac{2-\nu}{\nu}$), all borrowers who do not liquidate pay the same interim payment $p(\alpha) = p$ and owe the same amount, q , to the bank in period 2.*

The proof, which appears in the Appendix, has a simple structure. We conjecture that there must be at least two types that make different interim payments, and we find a contradiction. We focus on the highest two payments (and, by Lemma 2, these will correspond to the highest differing types). We find that borrowers at the ex-ante stage, when the menus are determined, would prefer that the top two pools be combined as a single pool in order to maximize their anticipated surplus. We use the distributional assumption on types at that stage since it allows us to quantify the ex-ante (at period 0) surplus. The uniform distribution helps in keeping this part of the analysis simple.

Since banks are perfectly competitive, the equilibrium outcome will indeed maximize the borrowers' surplus and so combine these top two pools. An induction argument for a finite number of pools will imply that one overall pool appears as the equilibrium contract. There are a number of different cases that must be considered (depending on the level of p and the size of the pools), but working through each of them is relatively straightforward.

Proposition 4 fully characterizes the structure of the equilibrium; however, to gain further insight and, in particular, to analyze welfare, we proceed by precisely calculating

the values of p and q , and the equilibrium liquidation policy.

3.3.1 Equilibrium payments and welfare

We begin by restating our notation to discuss the liquidation policy. Recall that l denotes the type that is “just indifferent” between liquidating and continuing the project with the (p, q) repayment schedule. Under perfect information, $l = 1 - \frac{Y}{2}$, which is the efficient level of liquidation; however, limited liability and the cross-subsidies between borrowers inside the pool (from higher-quality to lower-quality ones) will imply that $l < 1 - \frac{Y}{2}$. This reflects an important externality in our model. Whenever there is some pooling between borrowers, there will be cross-subsidies from higher-quality to lower-quality borrowers. This generates an inefficient liquidation policy, as some inefficient projects are not liquidated due to this implicit subsidy.

Proposition 3 allows us to focus on the case $l \leq p$. First, note that in the case that $l = 0$, it is trivial that the optimal choice of p is $p = 0$. Overall welfare in this case is $W = Y - 1$ (this is simply the average surplus generated by a project, given that all types of projects will be pursued).

Alternatively, it may be optimal to choose an interior l (i.e., between zero and one). In this case, we can characterize l by noting that two conditions must be satisfied. First, by definition, a borrower of type l must be indifferent between liquidating or continuing with the project; That is,

$$0 = \nu(B + l - q - r(p - l)). \quad (1)$$

In addition, banks need to break even, on average, so that the value of funds lent out is equal to the NPV of the project:

$$D = p + \nu q + (1 - \nu) \frac{1 + l}{2}. \quad (2)$$

Note that the indifference condition (1) implies that $B + \alpha > q$ for every $\alpha > l$, and so it is appropriate to write the break-even condition as above in (2), being sure that the loan will be fully repaid if the contract is successful for every borrowing type.

We can characterize the equilibrium p , under the assumption that both the optimal p and l are interior. Having done so, it is easy to verify conditions under which this is, indeed, the case and, then go on to consider outcomes when these conditions fail.

Continuing under the assumption that l is interior, we consider the first-order condition,

and we maximize total welfare in order to find the contract offered in the equilibrium. We begin with the expression of total welfare.

$$\begin{aligned}
W &= \int_l^1 (2x + \nu B - D)dx - (\nu r - 1) \int_l^p (p - x)dx \\
&= Y - 2 - l(Y - 2) - l^2 + 1 - \frac{1}{2}(\nu r - 1)(p - l)^2.
\end{aligned} \tag{3}$$

The first integral represents the net (positive or negative) welfare from each project financed, while the second integral is the welfare loss from inefficient borrowing. Note that the above expression (in the upper limit of the second integral) supposes that $p < 1$, which it will be easy to verify as true in equilibrium.

The first-order condition then characterizes the optimal p and bringing it together with (1) and (2) yields:

$$l = \frac{6\nu - 2Y - 2Y\nu + 8r\nu + \nu^2 - 4Yr\nu + 1}{6\nu + 8r\nu + \nu^2 + 1}, \text{ and} \tag{4}$$

$$p = \frac{6\nu - 4Y\nu + 8r\nu + \nu^2 - 4Yr\nu + 1}{6\nu + 8r\nu + \nu^2 + 1} \tag{5}$$

Substituting these expressions into equation (3), we can calculate a value W_I for welfare. The notation W_I is intended to highlight that this is the welfare under the optimal interior solution when it is feasible. However, this need not be the global optimum since choosing $p = 0 = l$ and generating an expected surplus of $Y - 1$ is always feasible.

Proposition 5 *When the worst possible project is sufficiently bad, then the optimal contract requires a strictly positive interim payment, and some projects are liquidated at the interim state. (Formally, both $l > 0$ and $p > 0$ when $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > Y$ and $W_I \geq Y - 1$.)*

Equations (4) and (5) show that in the parameter range where l is interior, both l and p are linear and decreasing in Y . That they should be decreasing is quite intuitive. As Y goes up, the least efficient and all other projects become more and more attractive, so the optimal first payment p goes down to decrease the liquidation threshold l . Conversely, as Y goes down, fewer projects should be funded, so p goes up.

Corollary 1 *There are parameter values for which, in equilibrium, there are borrowers who simultaneously borrow from both the bank and the hidden lender.*

Note that Corollary 1 is related to Proposition 3. However, whereas Proposition 3 shows that $l \leq p$, so that the lowest type to liquidate never consumes in the interim period, Corollary 1 demonstrates that when types are uniformly distributed, there are parameter values for which $l < p$. There are some borrowers who simultaneously borrow from both the bank and the hidden lender.

3.3.2 Effects on Simultaneous Borrowing and Debt Contracts

Bringing together all the results from the sections above, there are three equilibrium regimes. The diagram below illustrates these three equilibrium regions for a general ν .

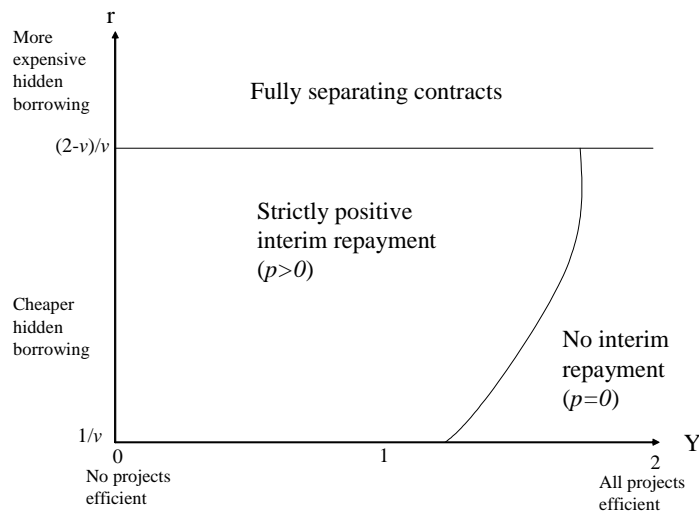


Figure 2: Different contractual forms as the efficiency of the hidden lender and NPV of the best project vary

In the absence of a hidden lender (or when its interest rate r is too high), banks can use interim payments to extract information about their borrowers. This information allows the banking sector to sort creditworthiness more effectively, with fully separating contracts that lead to efficient liquidation policies, as described in Proposition 1.

When the hidden lender is more efficient (r is lower), its presence alters the contracts

that the banks can feasibly sustain, as described in Proposition 4. The interest rate of loans does not vary with interim payments, and banks are less able to distinguish between types of borrowers. Two inefficiencies are at work. First, some borrowers simultaneously borrow from banks and the hidden lender; if the hidden lender is inefficient, this leads to a welfare loss. Second, there are cross-subsidies between borrowers, with higher-quality borrowers paying too large an interest rate for their bank loans and lower-quality borrowers getting interest rates below the break-even rate that would apply to them under perfect information. This leads to an inefficient liquidation policy; too many projects are funded and not liquidated. While a low (high) first payment improves (worsens) the inefficiency associated with borrowing from the hidden source, it makes the liquidation policies less (more) efficient. The chosen first payment, therefore, optimally solves the trade-off between these two inefficiencies. Depending on the efficiency of projects and the interest rate of the hidden lender, either an interior or a corner solution ($p = 0$) will be optimal. In particular, noting, from (5) that any strictly positive repayment is linear and decreasing in Y , then the equilibrium entails no interim repayment when projects are sufficiently efficient (Y is high enough) or strictly positive interim repayments which are strictly decreasing in Y .

In the next section, we explore in more detail the implications on outcomes and welfare of different levels of inefficiency of the hidden lender (as captured by r), and demonstrate that the level of interim repayments (in the case that pooling arises) are increasing in r .

3.3.3 Comparative statics

First, note that when $r > \frac{2-\nu}{\nu}$, welfare is first-best and independent of r within this range. In the case where the optimal contract involves $p = 0$, welfare is equal to $Y - 1$. Again, within this range, welfare is independent of r . Raising r to a level where either an interior p is optimal (which requires $W_I > Y - 1$) or the full separation equilibrium is attained (and the first-best level of welfare is achieved) trivially raises welfare.

The most interesting analysis is for parameters in the region with a single interior interim payment to the bank—that is, where $r \leq \frac{2-\nu}{\nu}$. Raising r so that the equilibrium shifts to the fully separating case, which is first-best, trivially raises welfare. We now consider how welfare varies with r within this region.

Having obtained explicit characterizations of l and p in terms of the exogenous parameters of the model and noting that welfare in this region is given by Equation (3), we consider the comparative statics of welfare. It is of particular interest to consider how welfare changes (and the channels through which it changes) as r , the exogenous rate of

interest in the opaque sector, varies.

First note that in the range $r < \frac{2-\nu}{\nu}$ and for $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > Y$ and $W_I \geq Y - 1$. We obtain the following.

Proposition 6 *The volume of loan continuation $(1-l)$ and the interim payment p required by the bank are decreasing in the hidden lender's rate r .*

These results are intuitive. As r increases it is more expensive for borrowers to hide inefficient projects and so they are more prone to liquidate. As a result, that l increases. Also, as r increases, the interim payment p need not be as high in order to deter borrowers from hiding inefficient projects and, since a high p imposes costs on some efficient borrowers, it should be reduced. Note that, these partial intuitions act in opposition to each other, but the result states that the overall effects on both l and p are these intuitive ones.

In particular, the first result suggests that one source of inefficiency is reduced since as r increases, l rises and so fewer inefficient projects are conducted. However, the second, in principle may have ambiguous welfare consequences: As r increases and the interim payment falls, and since the lowest type that borrows rises, the amount of borrowing from the opaque sector falls; but, the cost of borrowing from the opaque sector also rises. By examining welfare directly, we can see that the first of these two effects always dominates, as shown in Proposition 7.¹⁶

Proposition 7 *Welfare is always non-decreasing in the hidden lender's rate ($\frac{dW}{dr} \geq 0$). It is strictly increasing ($\frac{dW}{dr} > 0$) when the lender's rate is sufficiently low ($r < \frac{2-\nu}{\nu}$) and the proportion of inefficient projects is high ($2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > Y$ and $W_I > Y - 1$).*

3.4 Loan initiations

Within the model, borrowers only access the hidden lender to conceal their creditworthiness. In practice, hidden lenders fulfill various other roles. Their existence, in many cases, arises through their advantages in terms of provision of liquidity, enforcement technologies or different information sets. Whenever these advantages are important hidden lenders may substantially increase the availability of credit for some borrowers. At the same time,

¹⁶Note that the welfare as defined in Equation (3) does not take into account surplus gained by the alternative sector. Including this surplus in the welfare calculation would suggest that the only source of inefficiency would be inefficient liquidation, and so only the first effect would apply. The qualitative results would be unchanged—welfare increases in r . The analysis here would still be of interest, inasmuch as Equation (3) captures consumer surplus.

as discussed above, a hidden lender limits the average expected payoff of initiated projects. For this reason, if projects have an opportunity cost for the borrower, the presence of a hidden lender may, reduce the amount of loan initiations in the banking sector.

In the model of Section 2, since borrowers face no penalties for default and face no opportunity costs for taking out the loan in Period 0, all borrowers seek loans. The amount of borrowing at period 0 is unaffected by any of the parameters of the model, while the amount of hidden borrowing increases as r falls. However, this prediction changes if the borrower incurs an opportunity cost or otherwise puts something at risk in taking out a loan, then the effective overall cost of borrowing, which is relatively high when the hidden lending rate r is low, might put off some borrowers from borrowing or, indeed, forestall lending in the first place.

Suppose that, at period 0, each borrower has an opportunity cost C of initiating the investment project that is independent of the future α of the project. The borrower's expected payoff when taking on the loan is exactly the measure of welfare, W , analyzed in Section 3.3.3. A borrower would take out a loan at Period 0 if and only if $W \geq C$. Since W is non-decreasing in r by Proposition 7, a higher r would lead to more lending initiated at period 0.

Recall that a higher r leads to more liquidation and, so, proportionately less lending from the interim period. Overall, therefore, the effect of a higher cost of borrowing from the hidden sector has an ambiguous effect on the overall amount of lending and depends on whether the effect on loan initiation or on loan continuation is the more significant. In principle, one might be able to empirically distinguish these two effects inasmuch as they lead to to predictions about different stages in the lifetime of a loan.

3.5 Partially Hidden Borrowing

Next, we modify the model slightly to allow for a partially hidden lender. We introduce the possibility that the banking sector observes the level of hidden borrowing by the borrower with some probability $(1 - h)$. With probability h , borrowing from the non-banking sector remains hidden. A rationale for this modeling assumption is that the banking sector investigates each of its borrowers and obtains full information about the borrowing position of each with some probability $(1 - h)$.

Once a borrower is successfully investigated, her borrowing position with all possible alternative lenders is perfectly known by the whole banking sector. In this case, the banking sector will learn the borrower's type perfectly by viewing her borrowing position, and in

the continuation, full separation is achieved for sure. However, in case that the borrower is found to be borrowing d from the opaque sector, then she must incur a cost sd . This could be interpreted either as a penalty for hidden borrowing, or an early repayment fee to the hidden lender.¹⁷

Thus, the model with probabilistic observability of the hidden borrowing is like a switching model in which, with probability $(1 - h)$, full separation is achieved for sure, and, with probability h , the model looks like that of the previous sections. In this latter case, the only difference is that, from the borrower's point of view, the costs and benefits of the hidden borrowing need to be recalculated since, with probability $(1 - h)$, hidden borrowing is useless and entails a cost s .

In fact, once the alternative borrowing remains hidden, the rest of the model with probabilistic observation of the hidden borrowing can be fully solved by realizing that, in effect, the cost of borrowing from the hidden source is now $\frac{hr+(1-h)s}{h}$ instead of just r . Borrowing one unit from the hidden source costs r with probability h and costs s with probability $(1 - h)$. It only produces some concealment benefit to the borrower with probability h , so the whole cost has to be re-scaled by $1/h$.

We write $r(h, s) = \frac{hr+(1-h)s}{h}$ as the *effective* interest rate when borrowing from the opaque sector remains hidden with probability h ; the rate of interest is r when borrowing remains hidden; and the penalty cost, or cost of early repayment, when the banking sector observes the borrowing is s . With this notation, we obtain the following results, which are similar to those in the fully hidden case:

Proposition 8 *When the opaque sector lends at a sufficiently high effective interest rate ($r(h, s) > \frac{2-\nu}{\nu}$), then there exists a separating equilibrium with $p(\alpha) = \alpha$ and $q(\alpha) = \frac{D-\alpha-(1-\nu)\alpha}{\nu}$, and there is no borrowing from the hidden sector. When the opaque sector lends at a sufficiently low effective interest rate ($r(h, s) \leq \frac{2-\nu}{\nu}$), all types that do not liquidate make the same interim payment p and owe the same amount, q , to the bank in period 2.*

The functional form of the welfare equation and the incentive-compatibility conditions are similar to those of the basic model, so results similar to those of Section 3.3.1 hold. In particular, if $r(h, s) \leq \frac{2-\nu}{\nu}$, the only possible equilibrium is one with full pooling, and if

¹⁷In the latter case, $s < r - \frac{1}{\nu}$ is sufficient to guarantee that the agent would indeed prefer to repay the hidden borrower rather than investing or consuming those funds.

the optimal solution is interior, then total welfare is given by:

$$W = h \left[\int_l^1 (2x + vB - D) dx - (vr(h, s) - 1) \int_l^p (p - x) dx \right] + (1-h) \left[\int_{1-\frac{Y}{2}}^1 (2x + vB - D) dx \right], \quad (6)$$

where explicit expressions for l and p appear in the Appendix. Note that the expression in the first bracket is identical to the expression (3) in Section 3, with a change of the social cost of borrowing from r to $r(h, s)$, and that the second bracket is constant in p and l .

The welfare implications of the changes in the probability of the hidden sector becoming transparent ($1 - h$) are as follows: A higher ($1 - h$) implies higher welfare in a couple of ways. First is the automatic switching from the pooling equilibrium to the first-best full separation equilibrium whenever the banking sector observes the hidden lending. Second, increasing ($1 - h$) increases $r(h, s)$, and so the results on welfare increasing in r from Section 3 apply. Similarly, an increase in s raises $r(h, s)$ and so also raises welfare.

Note that our analysis is related to the literature on the interactions between direct screening of lenders through active investigation and the indirect screening that can be achieved by offering them a menu of contracts, as in Manove et al. (2001). While in most models these are seen as substitutes, in our model they are complements. That is, an increase in ($1 - h$) leads to more information about some borrowers directly and also to a more informative equilibrium with respect to the other borrowers (who may have loans from the alternative sector that remain hidden).¹⁸

Introducing, again, the concerns about borrower's opportunity cost, from Section 3.4, it is clear that the results on partially hidden borrowing—which act in much the same way as changing the efficiency of hidden borrowing—have natural implications for the volume of lending in the banking sector. In particular, the results of these two sections suggest that if it is easier for banks to observe lending from alternative lending sectors, the volume of bank lending initiated should increase.

¹⁸Even though, so far, we have considered h as an exogenous parameter, endogenizing it seems relatively straightforward. We could allow banks to choose their monitoring effort h at a cost. Higher transparency (lower h) would be more costly, and competition among banks should equalize the marginal cost of additional monitoring (reducing h) with its marginal gain in terms of borrowers' surplus in equilibrium.

4 Conclusions

While we presented a model of financing for an investment project, the central mechanisms and, in particular, the interaction of different sources of borrowing and the implications for contractual form have wide applicability. Our results highlight that one of the possible reasons that long-term debt contracts are inflexible with respect to interim payments is that the information that long-term lenders would extract from these interim payments would be corrupted by additional borrowing from hidden sources of funds. Our results also suggest an explanation for simultaneous borrowing from different sources, even when there is an apparently clear pecking order among them, and the borrowing from the cheaper source is not fully exhausted (for example, firm loans and trade credit or mortgages and credit card borrowing when both trade credit and credit card borrowing are not costlessly observable by the bank).

Existing literature has drawn a distinction between informal and formal lending and highlighted that the informal sector may increase credit availability through different information and enforcement technologies. In this paper we focus on opaque lending and, to the extent that informal lending may be opaque, we highlight an indirect channel through which it may diminish welfare. In particular through its effect on lending in the formal sector. When the indirect channel is strong enough to generate a net welfare loss, borrowers might (ex-ante) prefer to commit not to access informal lending, but with no means to do so, borrowers might find themselves compelled to access informal lending.

The model, and extensions, make predictions that, in principle, could be tested. Specifically, we highlight that changes in the efficiency or observability of alternative lending sources affect the form and extent of bank lending. The results on contractual form in the banking sector (moving from menus of contracts to a single interim payment) suggest that, as the informational transparency of the financial sector as a whole improves, banks are able to offer more-sophisticated financial instruments. The model also suggests that transparency and inefficiency of alternative lending lead to a higher volume of bank loans initiated. Finally, less efficient or more transparent alternative lending also suggest less (interim) liquidation of projects and a lower observed cost of bank funds.

Most of the empirical predictions of the model relate to the levels of efficiency and informational transparency of alternative lenders. Cross-country comparisons show substantial differences in the effective level of information sharing across countries (Miller, 2003; Jentzsch, 2007). In some countries, such as France, restrictions such as privacy

protection laws have precluded the creation of credit bureaus. In others, the existence of cheap-to-access and centralized public credit registers (that do not cover borrowing sources such as small credits, credit cards or consumer credit) has also crowded private ones. The model predicts that these differences should affect debt market contracts. Along these lines, it is suggestive that a higher level of innovations in mortgage markets in Anglo-Saxon countries has not had a counterpart in continental Europe (as suggested in Green and Wachter, 2005).

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A Omitted Proofs

Proof of Proposition 1

Proof. In order to characterize the equilibrium, we can draw on the revelation principle at $t = 1$ and think of the borrower’s choice from the menu $\{p, q(p)\}$ as a function of her type—that is, we could think of offering a menu $\{p(\alpha), q(\alpha)\}$.

Formally, the Proposition claims that $p(\alpha) = \alpha$, $q(\alpha) = \frac{D - \alpha - (1 - \nu)\alpha}{\nu}$, and that all types $\alpha < 1 - \frac{\nu}{2}$ liquidate at $t = \frac{1}{2}$. This last follows since the marginal type that liquidates is indifferent

between liquidating and receiving 0, or continuing the project and expecting a payoff of $\nu(B + \alpha - q(\alpha)) = \nu(B + \alpha - \frac{D - \alpha - (1-\nu)\alpha}{\nu}) = \nu B - D + 2\alpha = Y - 2 + 2\alpha$.

Turning to the characterization of $p(\alpha)$ and $q(\alpha)$: As discussed above, Assumption 4, together with competition among banks, ensures that any meaningful contract on the menu—that is, any contract that is ever taken up in equilibrium—will break even at each stage of the contract and so will not contain any observable cross-subsidies. The break-even condition, given that the first payment $p = \alpha$ reveals the type of the borrower as α , is that $D = \alpha + \nu q + (1 - \nu)\alpha$, so that in expectation the bank recovers its investment. This determines that the break-even second payment is $q = \frac{D - p - (1-\nu)p}{\nu}$.

Further, incentive-compatibility must be satisfied; that is, a borrower of type α prefers to make a first period payment $p(\alpha)$ than any other $p(\alpha')$. We analyze the incentive-compatibility condition by considering two deviations: imitating a lower type and imitating a higher type.

Incentive-compatibility condition 1: The contract needs to guarantee that no borrower wants to imitate a lower-quality borrower. Suppose that a borrower of quality α claims to be a lower-quality borrower $\alpha' < \alpha$ by paying a first payment $p = \alpha'$; in that case, her total utility would be $(\alpha - \alpha') + \nu(B - \frac{D - \alpha' - (1-\nu)\alpha'}{\nu} + \alpha)$. Note that $(\alpha - \alpha')$ is the additional consumption at $t = 1$ from reporting a lower type, while $(B - \frac{D - \alpha' - (1-\nu)\alpha'}{\nu} + \alpha)$ is the net consumption in the good state (which occurs with probability ν) after repaying $q(\alpha')$. Instead, by revealing her own type, she would get $\nu(B - \frac{D - \alpha - (1-\nu)\alpha}{\nu} + \alpha)$. The difference between these two terms is $-(1 - \nu)(\alpha - \alpha') < 0$, and so it cannot be optimal to claim to be a borrower of a lower type.

Incentive-compatibility condition 2: The contract also needs to guarantee that no borrower wants to imitate a higher-quality borrower by borrowing from the hidden source and paying a first payment $p > \alpha$. Suppose, for contradiction, that a borrower claims to be a higher-quality borrower by paying a first payment $p = \alpha'' > \alpha$ and borrowing $\alpha'' - \alpha$ from the hidden source to fund this payment. The total utility of the borrower would be $\nu(B - \frac{D - \alpha'' - (1-\nu)\alpha''}{\nu} - r(\alpha'' - \alpha) + \alpha)$ instead of $\nu(B - \frac{D - \alpha - (1-\nu)\alpha}{\nu} + \alpha)$. The difference between the two is:

$$(2 - v - vr)(\alpha'' - \alpha), \quad (7)$$

which is negative if and only if $r > \frac{2-\nu}{\nu}$, so this is the necessary and sufficient condition for this incentive-compatibility condition to hold. ■

Proof of Lemma 1

Proof. In the first-best, a borrower should be funded if and only if the expected NPV of the project is positive—that is, if and only if $-D + \alpha + v(B + \alpha) + (1 - \nu)\alpha \geq 0$.

In the equilibrium described above banks perfectly sort borrowers and offer break even deals so borrowers fully internalize the proceeds of their projects. Therefore the marginal borrower is precisely the one with NPV=0. ■

Proof of Lemma 2

Proof. Suppose that borrowers face the choice between two generic contracts a and b and without loss of generality, we label them so that $p_a > p_b$. The following possibilities are exhaustive:

- (i) $\alpha > p_a > \beta > p_b > \gamma$
- (ii) $p_a > p_b > \alpha > \beta > \gamma$
- (iii) $\alpha > \beta > \gamma > p_a > p_b$

(iv) $\alpha > p_a > p_b > \beta > \gamma$

(v) $\alpha > \beta > p_a > p_b > \gamma$

In cases (ii), (iii) and (v), the conditions for a borrower of type α to prefer a repayment of schedule a to one of type b are identical to the conditions for a borrower of type β . It remains to consider cases of type (i) and (iv).

In Case (i) a borrower of type β prefers schedule a to schedule b whenever

$$\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu) + r\nu(p_a - \beta), \quad (8)$$

and a borrower of type α prefers schedule b to schedule a whenever the following condition is satisfied:

$$\alpha - p_b + \nu(B + \alpha - p_b - q_b) \geq \alpha - p_a + \nu(B + \alpha - p_a - q_a), \quad (9)$$

or, equivalently, $\nu(q_b - q_a) \leq (p_a - p_b)(1 + \nu)$, which contradicts (8).

Finally, in Case (iv), the condition for a type α borrower to prefer the b schedule is $\nu(q_b - q_a) \leq (p_a - p_b)(1 + \nu)$, and the condition for a type β borrower to prefer the a schedule is that $\nu(q_b - q_a) \geq (p_a - p_b)(1 + \nu + r\nu)$. These conditions are mutually incompatible.

In all cases, therefore, it cannot be that a borrower of type $\alpha > \beta$ strictly prefers the schedule with the first payment $p_b < p_a$ and the borrower of type β prefers the schedule with the first payment p_a . This completes the proof. ■

Proof of Proposition 2

Proof. To show that with $r < \frac{2-\nu}{\nu}$ there cannot be an equilibrium where a continuum of borrowers are able to separate, we proceed in a similar fashion as with the proof of Proposition 1 and show that if two borrowers that are arbitrarily close to each other are able to separate, we reach a contradiction.

We start by conjecturing an equilibrium menu that achieves the separation of some borrowers in a continuum and then pick two arbitrarily close borrowers α and α' with $\alpha < \alpha'$ and $p(\alpha) \neq p(\alpha')$. The corresponding break-even second payments $q(\alpha) = \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu}$ and $q(\alpha') = \frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu}$. We know by Lemma 2 that $p(\alpha) < p(\alpha')$. These payment schedules have to fulfill similar incentive-compatibility conditions to the ones shown in Proposition 1.

In particular, we can define the two conditions as:

IC1: no borrower of a higher type (α') wants to imitate a borrower of a lower type (α).

IC2: No borrower of a lower type (α) wants to imitate a borrower of a higher type (α').

If there is a continuum of borrowers that can individually separate, at least one of the following situations must be true.

a) At least two arbitrarily close borrowers are neither consuming nor borrowing from a hidden lender at $t = 1$

b) At least two arbitrarily close borrowers are both consuming $t = 1$

c) At least two arbitrarily close borrowers are both borrowing from a hidden lender at $t = 1$

We analyse each of this situations in turn.

a) This part of the equilibrium is characterized by Proposition 1, and we know that IC2 cannot hold in this situation if $r < \frac{2-\nu}{\nu}$.

b) Suppose that there is a borrower α' that fully separates from the rest and is able to consume at $t = 1$ (that is $p(\alpha') < \alpha'$). Then there must be a borrower α , such that $\alpha < \alpha'$, that is also able to pay $p(\alpha')$ without borrowing. The utility of borrower α of claiming his own type is $\nu(B - \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu} +$

$\alpha) + (\alpha - p(\alpha))$ and the utility of imitating borrower α' is $\nu(B - \frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu} + \alpha) + (\alpha - p(\alpha'))$. The necessary and sufficient condition for IC2 to hold is therefore:

$$\nu(B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu} + \alpha) + (\alpha - p(\alpha)) > \nu(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} + \alpha) + (\alpha - p(\alpha')),$$

which simplifies to: $(1 - \nu)(\alpha - \alpha') > 0$, which is always false, so we reach a contradiction.

c) In this case we start by exploring IC2.

A borrower of a lower type would have a utility of $\nu((B - \frac{D-p(\alpha)-(1-\nu)\alpha}{\nu} + \alpha) - r(p(\alpha) - \alpha))$, while claiming to be a higher-type borrower would yield her a utility of $\nu(B - \frac{D-p(\alpha')-(1-\nu)\alpha'}{\nu} + \alpha - r(p(\alpha') - \alpha))$. Subtracting the first term from the second we get a condition that must be smaller than zero for IC2 to hold.

$$\nu(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} - r(p(\alpha') - \alpha) + \alpha) - \nu((B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu}) - r(p(\alpha) - \alpha) + \alpha) < 0,$$

which can be simplified as $(1 - \nu)(\alpha' - \alpha) + (1 - r\nu)(p(\alpha') - p(\alpha)) < 0$.

However, in this case IC1 becomes:

$$\nu(B - \frac{D - p(\alpha') - (1 - \nu)\alpha'}{\nu} - r(p(\alpha') - \alpha') + \alpha') - \nu((B - \frac{D - p(\alpha) - (1 - \nu)\alpha}{\nu}) - r(p(\alpha) - \alpha') + \alpha') > 0.$$

This expression simplifies to $(1 - \nu)(\alpha' - \alpha) + (1 - r\nu)(p(\alpha') - p(\alpha)) > 0$ which is exactly the opposite condition to the one necessary for IC2. Therefore, when two arbitrarily close borrowers borrow and achieve separation, IC1 and IC2 are mutually incompatible, which poses a contradiction.

This last part of the proposition follows by noting that Lemma 2 implies that for every three borrowers with types α , β , and γ such that $\alpha > \beta > \gamma$ where $p(\alpha) = p(\gamma)$, it must be the case that $p(\alpha) = p(\beta) = p(\gamma)$. ■

Proof of Proposition 3

Proof. By contradiction. Conditional on $l > p$, the utility of the indifferent borrower l can be expressed as $\nu(B - q(l) + l + (l - p(l)))$. Given that liquidating provides utility equal to zero and that the borrower is indifferent, this implies that

$$\nu(B + l - q(l) + (l - p(l))) = 0. \tag{10}$$

As $l > p$, then $(l - p(l)) > 0$. This implies jointly with (10) that $B + l - q(l) < 0$, which violates Assumption 5. ■

Proof of Proposition 4

Proof. We prove by contradiction. Suppose that this result is false; then there must be at least two types that pay different amounts. We focus on the highest two payments (and by Lemma 2 these will correspond to the highest differing types). We will find that in equilibrium, the top two pools would rather be combined as a single pool. Then, an induction argument for a finite number of pools will imply that one overall pool appears as the equilibrium contract, as there cannot be any top two pools.

We continue by considering the top two pools of types that do not liquidate.

First note that if any type α strictly prefers not to liquidate, then all types $\beta > \alpha$ would prefer to mimic α than to liquidate. Thus, in restricting attention to the highest two payments $p_1 < p_2$ and associated types (and by Lemma 2 we know that higher types are associated with higher payments), we can be sure that there are some $\alpha_1 \leq \alpha_2$ such that types $(\alpha_1, \alpha_2]$ pay p_1 in the first period (with the associated q_1) and $(\alpha_2, 1]$ pay p_2 in the first period (with the associated q_2 in the second period).

The resulting contradiction is somewhat involved, but the structure is as follows. First, we highlight a number of possible cases. In each case, we seek to determine the optimal choice of α_2 (and associated p_1 and p_2) given that there are two pools in the range $(\alpha_1, 1]$ while keeping α_1 indifferent (and so all other types below may also remain with their existing contracts, and there are no changes to equilibrium or welfare consequences from types below α_1).

By definition, $p_2 > p_1$. There are a number of cases to consider:

- I $p_2 \geq 1$ and $p_1 \geq \alpha_2$
- II $p_2 \geq 1$ and $\alpha_2 \geq p_1 \geq \alpha_1$
- III $p_2 \geq 1$ and $\alpha_1 \geq p_1$
- IV $1 \geq p_2 \geq \alpha_2$ and $p_1 \geq \alpha_2 \geq \alpha_1$
- V $1 \geq p_2 \geq \alpha_2$ and $\alpha_2 \geq p_1 \geq \alpha_1$
- VI $1 \geq p_2 \geq \alpha_2$ and $\alpha_1 \geq p_1$

Focusing on each case in turn, it is tedious but straightforward to show that the optimum outcome in all cases pushes α_2 into a corner.¹⁹ This necessarily implies that the equilibrium α_2 that maximizes welfare is such that $\alpha_2 \in \{\alpha_1, 1, p_2\}$. The first two options $\alpha_2 \in \{\alpha_1, 1\}$ contradict the assumption that there are two distinct pools of borrowers. To complete the proof we finally suppose that $\alpha_2 = p_1$. There are two remaining possibilities to consider, either $p_2 > 1$ or $1 > p_2 > \alpha_2$, in both cases $p_2 > \alpha_2$. We show that this is inconsistent with the maintained assumption that $1 > \alpha_2 > \alpha_1$.

When $p_2 > \alpha_2 > \alpha_1$, the incentive-compatibility condition for a borrower of type α_2 is given by

$$\nu(B + \alpha_2 - q_2 - r(p_2 - \alpha_2)) = (\alpha_2 - p_1) + \nu(B + \alpha_2 - q_1),$$

which yields $p_2 = \alpha_2 + \frac{(1-\nu)(1-\alpha_1)}{2(r\nu-1)}$. The constraint for a borrower of type α_1 is

$$\nu(B + \alpha_1 - q_1 - r(p_1 - \alpha_1)) = k. \tag{11}$$

Substituting for q_1 and since $p_1 = \alpha_2$, we obtain:

$$\alpha_2 = \frac{2B\nu - 2D - 2k + \alpha_1 + \nu\alpha_1 + 2r\nu\alpha_1}{\nu + 2r\nu - 3}. \tag{12}$$

The case is not degenerate—that is, in equilibrium these top two pools do not collapse into one—as long as α_2 is interior. In particular, it must be that both $\alpha_2 > \alpha_1$ and $1 > \alpha_2$. Specifically, substituting from (12) and rearranging $\alpha_2 > \alpha_1$ if and only if $\alpha_1 > \frac{1}{2}(1 + k)$. Similarly, $\alpha_2 < 1$ if and only if $\alpha_1 < \frac{\nu + 2r\nu - 1 + 2k}{(1 + \nu + 2r\nu)}$. For an interior solution $1 > \alpha_2 > \alpha_1$, both conditions must hold and in particular:

$$\frac{\nu + 2r\nu - 1 + 2k}{(1 + \nu + 2r\nu)} > \frac{1}{2}(1 + k), \tag{13}$$

¹⁹Full details are available from the authors. Case II is somewhat more involved inasmuch as it involves examination of the second order condition.

rearranging this is true if and only if $k > 1$. This is impossible—the highest possible utility for a borrower is for the best possible type (type 1) to be recognized as such, and in this case her expected utility would be $\nu B - D + 1 + 1 = 1$ and so it cannot be that k , which is the expected utility for the α_1 type, is greater than 1, providing the contradiction.

By induction, if the top two pools cannot exist, the only possible equilibrium with a finite number of pools is one with only one pool. ■

Proof of Proposition 5

Proof. Note that p is linear in Y . It is sufficient, therefore, to consider the two extremes $Y = 2$ and $Y = 0$. For $Y = 2$, $p = \frac{(1-\nu)^2}{6\nu+8r\nu+\nu^2+1}$ which is greater than 0 and less than 1. Furthermore, for $Y = 0$, $p = 1$ and $l > 0$ as long as $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > Y$.

For values of Y higher than $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)}$, the optimal contract is to set $p = 0$, which leads to $l = 0$.

Thus, $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > Y$ is required for an interior l and p to be feasible. An interior l would generate more surplus and so would be the equilibrium outcome when the welfare generated is higher than the next best alternative—choosing $l = 0$ and $p = 0$, or, equivalently, $W_I \geq Y - 1$.

Note that for $Y = 0$, both $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+2r\nu)} > Y$ and $W_I \geq Y - 1$ hold as strict inequalities, and so, in particular, for small enough Y , both l and p will be interior.

Finally, as Y decreases, the parameters are more likely to be such that the optimal choice of l is interior; that is, $W_I + 1 - Y$ is decreasing in Y . To prove this, first note $W_I \geq Y - 1$ if and only if

$$A = \frac{2(Y-2)(6\nu+8r\nu+\nu^2+1)(2Y-6\nu+2Y\nu-8r\nu-\nu^2+4Yr\nu-1)}{-2(6\nu-2Y-2Y\nu+8r\nu+\nu^2-4Yr\nu+1)^2-(\nu r-1)(2Y-16\nu+6Y\nu)^2} \geq 0 \quad (14)$$

Taking the derivative with respect to Y yields

$$\frac{dA}{dY} = -2(6\nu+8r\nu+\nu^2+1)(6\nu-4Y\nu+8r\nu+\nu^2-4Yr\nu+1) \quad (15)$$

Note that this expression is linear in Y and so $\frac{dA}{dY}$ takes its maximal value when $Y = 2$ when this value is

$$\frac{dA}{dY} = -2(6\nu+8r\nu+\nu^2+1)(1-\nu)^2, \quad (16)$$

which it can be easily verified is negative in the range $\nu \in (0, 1)$.

In particular, it is always the case that $W_I \geq Y - 1$ when $Y = 0$. ■

Proof of Corollary 1

Proof. Proposition 5 states that there are parameter values for which l and p are interior. In this case, l and p take values as given by expressions (5) and (4). Note that $p > l$ since the expression $\frac{6\nu-4Y\nu+8r\nu+\nu^2-4Yr\nu+1}{6\nu+8r\nu+\nu^2+1} > \frac{6\nu-2Y-2Y\nu+8r\nu+\nu^2-4Yr\nu+1}{6\nu+8r\nu+\nu^2+1}$ can be simplified as $Y(1-\nu) > 0$, which is always true. Since $p > l$, borrowers need to borrow from the hidden source to satisfy the first payment. ■

Proof of Proposition 6

Proof. Follows by deriving

$$\frac{dl}{dr} = \frac{4\nu(3+\nu)(1-\nu)Y}{(6\nu+8r\nu+\nu^2+1)^2} > 0, \text{ and} \quad (17)$$

$$\frac{dp}{dr} = -\frac{4\nu(1-\nu)^2Y}{(6\nu+8r\nu+\nu^2+1)^2} < 0 \quad (18)$$

from (4) and (5). ■

Proof of Proposition 7

Proof. We begin by considering the parameter range $r < \frac{2-\nu}{\nu}$, $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+r\nu)} > Y$ and $W_I > Y - 1$. In this range $W = W_I$.

Note that $p-l = \frac{2(1-\nu)Y}{6\nu+8r\nu+\nu^2+1}$ and so $\frac{d(p-l)}{dr} = -\frac{16\nu(1-\nu)Y}{(6\nu+8r\nu+\nu^2+1)^2}$. Next, by taking the derivative of W with respect to r from Equation 3, we obtain:

$$\frac{dW}{dr} = \frac{dl}{dr}(2-Y-2l) - \frac{1}{2}\nu(p-l)^2 - (\nu r - 1)(p-l)\frac{d(p-l)}{dr}. \quad (19)$$

Substituting in for $\frac{dl}{dr}$ and $\frac{d(p-l)}{dr}$ and simplifying:

$$\frac{dW}{dr} = \frac{4\nu + 16\nu^2 - 40\nu^3 + 16\nu^4 + 4\nu^5 + 32r\nu^2 - 64r\nu^3 + 32r\nu^4}{2(6\nu + 8r\nu + \nu^2 + 1)^3} Y. \quad (20)$$

The denominator of this expression is positive and so $\frac{dW}{dr}$ has the same sign as the numerator of the fraction. Specifically,

$$\begin{aligned} \text{sign}\left(\frac{dW}{dr}\right) &= \text{sign}(4\nu + 16\nu^2 - 40\nu^3 + 16\nu^4 + 4\nu^5 + 32r\nu^2 - 64r\nu^3 + 32r\nu^4) \\ &= \text{sign}(1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4 + 8r\nu - 16r\nu^2 + 8r\nu^3) \end{aligned}, \quad (21)$$

where the second equality holds, since the sign of the factor (4ν) is positive. It follows that $\frac{dW}{dr} > 0$ if and only if $1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4 + 8r\nu - 16r\nu^2 + 8r\nu^3 > 0$, which is true if and only if:

$$\frac{1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4}{16\nu^2 - 8\nu - 8\nu^3} > r. \quad (22)$$

Note that $\frac{2-\nu}{\nu} > r \geq \frac{1}{\nu}$ and so $\frac{dW}{dr} > 0$ requires

$$\frac{1 + 4\nu - 10\nu^2 + 4\nu^3 + \nu^4}{16\nu^2 - 8\nu - 8\nu^3} > \frac{1}{\nu}, \quad (23)$$

or, equivalently,

$$4\nu^3 - 2\nu^2 - 12\nu + \nu^4 + 9 > 0, \quad (24)$$

which is always true for ν in the range $(0, 1)$.

Outside of the parameter range $r < \frac{2-\nu}{\nu}$, $2 - \frac{(3+\nu)(1-\nu)}{2(1+\nu+r\nu)} > Y$ and $W_I > Y - 1$, $\frac{dW}{dr} = 0$ trivially since W is independent of r . ■

Interim payment and liquidation under partially hidden borrowing

The optimal first payment is:

$$p = \frac{\nu^2 + 1 - 4(Y - 2)\nu - 2\nu - 4(Y - 2)r(h, s)\nu}{6\nu + 8r(h, s)\nu + \nu^2 + 1} \quad (25)$$

and the type of borrower who is just indifferent between liquidating the project and continuing it is:

$$l = \frac{2\nu - 2(Y - 2) - 2(Y - 2)\nu + 4r(h, s) + \nu^2 - 3}{6\nu + 8r(h, s)\nu + \nu^2 + 1}. \quad (26)$$

Total welfare in this regime can be expressed as:

$$W = h \left[\int_l^1 (2x + vB - D)dx - (\nu r - 1) \int_l^p (p - x)dx \right] + (1 - h) \left[\int_{1 - \frac{Y}{2}}^1 (2x + vB - D)dx - \nu s \int_l^p (p - x)dx \right]. \quad (27)$$

Proof of Proposition 8

Proof. The proof is almost identical to the ones in Propositions 1 and 4, except that, now, borrowing from the hidden source entails higher costs, and so further details are omitted. ■